A renormalisable non-anticommutative supersymmetric SU(N)⊗U(1) gauge theory in components

A renormalisable non-anticommutative supersymmetric $\mathrm{SU}(N) \otimes \mathrm{U}(1)$ gauge theory in components

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ABSTRACT: We discuss the non-anticommutative ($\mathcal{N} = \frac{1}{2}$) supersymmetric $\mathrm{SU}(N) \otimes \mathrm{U}(1)$ gauge theory including a superpotential. We show how recent proposals for obtaining a renormalisable version of the theory may be implemented in the component formalism at the one-loop level.

Contents

1 Introduction

Deformed quantum field theories have been subject to renewed attention in recent years due to their natural appearance in string theory. Initial investigations focussed on theories on non-commutative spacetime in which the commutators of the spacetime co-ordinates become non-zero. More recently [1–9], non-anticommutative supersymmetric theories have been constructed by deforming the anticommutators of the Grassmann co-ordinates θ^α (while leaving the anticommutators of the $\overline{\theta}^{\dot\alpha}$ unaltered). Consequently, the anticommutators of the supersymmetry generators $\overline{Q}_{\dot\alpha}$ are deformed while those of the Q_α are unchanged. It is straightforward to construct non-anticommutative versions of ordinary supersymmetric theories by taking the superspace action and replacing the ordinary product by the Moyal $*$-product [10] which implements the non-anticommutativity. Non-anticommutative versions of the Wess-Zumino model and supersymmetric gauge theories have been formulated in four dimensions [10, 11] and their renormalisability discussed [12–17], with explicit computations up to two loops [18] for the Wess-Zumino model and one loop for gauge theories [19–23]. Even more recently, non-anticommutative theories in two dimensions have been constructed [24–28], and their one-loop divergences computed [29, 30]. In ref. [31] we returned to a closer examination of the non-anticommutative Wess-Zumino model (with a superpotential) in four dimensions, and showed that to obtain correct results for the theory where the auxiliary fields have been eliminated, from the corresponding results for the uneliminated theory, it is necessary to include in the classical action separate couplings for all the terms which may be generated by the renormalisation process; and finally in ref. [32] we extended this analysis to the gauged U(1) case.

In ref. [23] we considered the renormalisation of an $\mathcal{N} = \frac{1}{2}$ theory with a superpotential (for the case of adjoint matter) and with a mass term (for the case of matter in the fundamental and anti-fundamental representations); note that $\mathcal{N} = \frac{1}{2}$ supersymmetry does not allow a trilinear term in the latter case. We found there were obstacles to obtaining a renormalisable theory with a superpotential in the adjoint case. The requirements of

$\mathcal{N} = \frac{1}{2}$ invariance and renormalisability impose the choice of gauge group $\mathrm{SU}(N) \otimes \mathrm{U}(1)$ (rather than $\mathrm{SU}(N)$ or $\mathrm{U}(N)$) [19, 20]. In the adjoint case with a trilinear superpotential, the matter fields must also be in a representation of $\mathrm{SU}(N) \otimes \mathrm{U}(1)$. The problem is that the potential part of the classical action contains terms with different combinations of $\mathrm{SU}(N)$ and $\mathrm{U}(1)$ chiral fields which mix under $\mathcal{N} = \frac{1}{2}$ supersymmetry, but for which the Yukawa couplings renormalise differently, at least in the simplest version of the theory. However, recently an elegant solution to this problem has been proposed [33] in which the kinetic terms for the $\mathrm{U}(1)$ chiral fields are modified, in such a way that the $\mathrm{SU}(N)$ and $\mathrm{U}(1)$ chiral fields (and consequently their Yukawa couplings) renormalise in exactly the same way. The authors of ref. [33] worked in superspace; our purpose here is to confirm that a similar procedure can be carried out in the component formalism.

2 The classical adjoint action

In this section we present the classical form of the adjoint $\mathcal{N} = \frac{1}{2}$ action with a superpotential in the component formalism, including the modifications suggested in ref. [33]. The adjoint action was first introduced in ref. [11] for the gauge group $\mathrm{U}(N)$. However, as we noted in refs. [19, 20], at the quantum level the $\mathrm{U}(N)$ gauge invariance cannot be retained since the $\mathrm{SU}(N)$ and $\mathrm{U}(1)$ gauge couplings renormalise differently; and we are obliged to consider a modified $\mathcal{N} = \frac{1}{2}$ invariant theory with the gauge group $\mathrm{SU}(N) \otimes \mathrm{U}(1)$. In the adjoint case with a Yukawa superpotential, it turns out that the matter fields must also be in the adjoint representation of $\mathrm{SU}(N) \otimes \mathrm{U}(1)$. The classical action with a superpotential may be written

$$
\begin{aligned}
S_0 = \int d^4 x \Bigg\{ & e^{AB} \left(-\frac{1}{4} F^{\mu\nu A} F^B_{\mu\nu} - i\overline{\lambda}^A \overline{\sigma}^\mu (D_\mu \lambda)^B + \frac{1}{2} D^A D^B \right) \\
& -\frac{1}{2} i C^{\mu\nu} d^{ABC} e^{AD} F^D_{\mu\nu} \overline{\lambda}^B \overline{\lambda}^C \\
& +\overline{F}F - i\overline{\psi}\overline{\sigma}^\mu D_\mu \psi - D^\mu \overline{\phi} D_\mu \phi + \overline{\phi} D^F \phi + i\sqrt{2}(\overline{\phi}\lambda^F \psi - \overline{\psi}\overline{\lambda}^F \phi) \\
& +C^{\mu\nu}(\sqrt{2} D_\mu \overline{\phi} \lambda^D \overline{\sigma}_\nu \psi + i\overline{\phi} F^D_{\mu\nu} F) \\
& +(\kappa - 1)\big[\overline{F}^0 F^0 - i\overline{\psi}^0 \overline{\sigma}^\mu \partial_\mu \psi^0 - \partial^\mu \overline{\phi}^0 \partial_\mu \phi^0 \\
& +d^{000} C^{\mu\nu}(\sqrt{2} \partial_\mu \overline{\phi}^0 \overline{\lambda}^0 \overline{\sigma}_\nu \psi^0 + i\overline{\phi}^0 F^0_{\mu\nu} F^0) \\
& +d^{ab0} C^{\mu\nu}(\sqrt{2} D_\mu \overline{\phi}^a \overline{\lambda}^b \overline{\sigma}_\nu \psi^0 + i\overline{\phi}^a F^b_{\mu\nu} F^0) \big] \\
& +\frac{1}{2}\left(y d^{ABC} \phi^A \phi^B F^C - y d^{ABC} \phi^A \psi^B \psi^C + \overline{y} d^{ABC} \overline{\phi}^A \overline{\phi}^B \overline{F}^C - \overline{y} d^{ABC} \overline{\phi}^A \overline{\psi}^B \overline{\psi}^C \right) \\
& +\frac{1}{3} i\overline{y} C^{\mu\nu} f^{abc} D_\mu \overline{\phi}^a D_\nu \overline{\phi}^b \overline{\phi}^c - \frac{1}{3} i\overline{y} C^{\mu\nu} d^{ABE} d^{CDE} F^D_{\mu\nu} \overline{\phi}^A \overline{\phi}^B \overline{\phi}^C \\
& +\kappa_1 \sqrt{2} C^{\mu\nu} d^{abc}(\overline{\phi}^a \overline{\lambda}^b \overline{\sigma}_\nu D_\mu \psi^c + D_\mu \overline{\phi}^a \overline{\lambda}^b \overline{\sigma}_\nu \psi^c + i\overline{\phi}^a F^b_{\mu\nu} F^c) \\
& +\kappa_2 \sqrt{2} C^{\mu\nu} d^{ab0}(\overline{\phi}^0 \overline{\lambda}^a \overline{\sigma}_\nu D_\mu \psi^b + \partial_\mu \overline{\phi}^0 \overline{\lambda}^a \overline{\sigma}_\nu \psi^b + i\overline{\phi}^0 F^a_{\mu\nu} F^b) \\
& +\kappa_3 \sqrt{2} C^{\mu\nu} d^{ab0}(\overline{\phi}^a \overline{\lambda}^b \overline{\sigma}_\nu \partial_\mu \psi^0 + D_\mu \overline{\phi}^a \overline{\lambda}^b \overline{\sigma}_\nu \psi^0 + i\overline{\phi}^a F^b_{\mu\nu} F^0) \\
& +\kappa_4 \sqrt{2} C^{\mu\nu} d^{0ab}(\overline{\phi}^a \overline{\lambda}^0 \overline{\sigma}_\nu D_\mu \psi^b + D_\mu \overline{\phi}^a \overline{\lambda}^0 \overline{\sigma}_\nu \psi^b + i\overline{\phi}^a F^0_{\mu\nu} F^b) \\
& +\kappa_5 \sqrt{2} C^{\mu\nu} d^{000}(\overline{\phi}^0 \overline{\lambda}^0 \overline{\sigma}_\nu \partial_\mu \psi^0 + \partial_\mu \overline{\phi}^0 \overline{\lambda}^0 \overline{\sigma}_\nu \psi^0 + i\overline{\phi}^0 F^0_{\mu\nu} F^0) \Bigg\}.
\end{aligned}
\tag{2.1}
$$

where

$$\lambda^F = \lambda^A \tilde{F}^A, \qquad\qquad (\tilde{F}^A)^{BC} = if^{BAC},$$
$$\lambda^D = \lambda^A \tilde{D}^A, \qquad\qquad (\tilde{D}^A)^{BC} = d^{ABC}, \qquad (2.2)$$

(similarly for D^F, $F^D_{\mu\nu}$), and we have

$$D_\mu \phi = \partial_\mu \phi + iA^F_\mu \phi,$$
$$F^A_{\mu\nu} = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu - f^{ABC}A^B_\mu A^C_\nu, \qquad (2.3)$$

with similar definitions for $D_\mu \psi$, $D_\mu \lambda$. If one decomposes U(N) as SU(N) $\otimes$ U(1) then our convention is that ϕ^a (for example) are the SU(N) components and ϕ^0 the U(1) component. Of course then $f^{ABC} = 0$ unless all indices are SU(N). We note that $d^{ab0} = \sqrt{\frac{2}{N}}\delta^{ab}$, $d^{000} = \sqrt{\frac{2}{N}}$. (Useful identities for U(N) are listed in the appendix.) We also have

$$e^{ab} = \frac{1}{g^2}, \quad e^{00} = \frac{1}{g_0^2}, \quad e^{0a} = e^{a0} = 0. \qquad (2.4)$$

Compared with our previous work such as ref. [23], we have absorbed a factor of g into our definitions of the fields in the gauge multiplet. For simplicity of exposition we shall omit (here and elsewhere) terms which are $\mathcal{N} = \frac{1}{2}$ supersymmetric on their own (such as terms involving only $\overline{\phi}$, $\overline{\lambda}$ and/or F). Such terms are present in the action as obtained by reduction of the superspace action to components, and they are also generated by quantum corrections even if omitted from the classical action; but they do not add to our understanding of the renormalisability of the theory, which is our main concern here. They were considered in full in refs. [33]; and indeed we included them ourselves in refs. [19, 20]. We have, however, taken the opportunity of including here some additional sets of terms (those multiplied by κ_{1-5}) which will be required for renormalisability of the theory. Each of these sets of terms is separately $\mathcal{N} = \frac{1}{2}$ invariant. Note that for the chiral field kinetic part of the action in eq. (2.1), $\overline{F}F \equiv \overline{F}^A F^A = \overline{F}^a F^a + \overline{F}^0 F^0$, etc; the U(1) part $\overline{F}^0 F^0$ could have been combined with that in the $(\kappa - 1)$ part of the action, as could the kinetic terms with ϕ^0 and ψ^0, with some attendant simplification. We have left the action in its present form to facilitate comparison with ref. [33].

It is easy to show that eq. (2.1) is invariant under

$$\delta A^A_\mu = -i\overline{\lambda}^A \overline{\sigma}_\mu \epsilon$$
$$\delta \lambda^A_\alpha = i\epsilon_\alpha D^A + (\sigma^{\mu\nu}\epsilon)_\alpha \left[F^A_{\mu\nu} + \frac{1}{2}iC_{\mu\nu}d^{ABC}\overline{\lambda}^B\overline{\lambda}^C \right], \quad \delta\overline{\lambda}^A_{\dot\alpha} = 0,$$
$$\delta D^A = -\epsilon\sigma^\mu D_\mu \overline{\lambda}^A,$$
$$\delta\phi = \sqrt{2}\epsilon\psi, \quad \delta\overline{\phi} = 0,$$
$$\delta\psi^\alpha = \sqrt{2}\epsilon^\alpha F, \quad \delta\overline{\psi}_{\dot\alpha} = -i\sqrt{2}(D_\mu \overline{\phi})(\epsilon\sigma^\mu)_{\dot\alpha},$$
$$\delta F^A = 0,$$
$$\delta\overline{F}^A = -i\sqrt{2}D_\mu \overline{\psi}^A \overline{\sigma}^\mu \epsilon - 2i(\overline{\phi}\epsilon\lambda^F)^A + 2C^{\mu\nu}D_\mu(\overline{\phi}^B \epsilon\sigma_\nu(\overline{\lambda}^D)^{AB}). \qquad (2.5)$$

In eq. (2.1), $C^{\mu\nu}$ is related to the non-anti-commutativity parameter $C^{\alpha\beta}$ by

$$C^{\mu\nu} = C^{\alpha\beta}\epsilon_{\beta\gamma}\sigma_\alpha^{\mu\nu\gamma}, \qquad (2.6)$$

where

$$\begin{aligned}
\sigma^{\mu\nu} &= \frac{1}{4}(\sigma^\mu\overline{\sigma}^\nu - \sigma^\nu\overline{\sigma}^\mu), \\
\overline{\sigma}^{\mu\nu} &= \frac{1}{4}(\overline{\sigma}^\mu\sigma^\nu - \overline{\sigma}^\nu\sigma^\mu).
\end{aligned} \qquad (2.7)$$

Our conventions are in accord with [10]; in particular,

$$\sigma^\mu\overline{\sigma}^\nu = -\eta^{\mu\nu} + 2\sigma^{\mu\nu}. \qquad (2.8)$$

Properties of C which follow from eq. (2.6) are

$$\begin{aligned}
C^{\alpha\beta} &= \frac{1}{2}\epsilon^{\alpha\gamma}\,(\sigma^{\mu\nu})_\gamma{}^\beta C_{\mu\nu}, \\
C^{\mu\nu}\sigma_{\nu\alpha\dot\beta} &= C_\alpha{}^\gamma \sigma^\mu{}_{\gamma\dot\beta}, \\
C^{\mu\nu}\overline{\sigma}_\nu^{\dot\alpha\beta} &= -C^\beta{}_\gamma \overline{\sigma}^{\mu\dot\alpha\gamma}.
\end{aligned} \qquad (2.9)$$

We use the standard gauge-fixing term

$$S_{\text{gf}} = \frac{1}{2\alpha}\int d^4x(\partial.A)^2 \qquad (2.10)$$

with its associated ghost terms. The vector propagator is given by

$$\Delta_{V\mu\nu}^{AB} = -\frac{1}{p^2}\left(\eta_{\mu\nu} + (\alpha-1)\frac{p_\mu p_\nu}{p^2}\right)\left(e^{-1}\right)^{AB}. \qquad (2.11)$$

The scalar propagator is

$$\Delta_\phi^{AB} = -\frac{1}{p^2}P^{AB} \qquad (2.12)$$

where

$$P^{ab} = \delta^{ab}, \quad P^{00} = \frac{1}{\kappa}, \quad P^{0a} = P^{a0} = 0, \qquad (2.13)$$

the fermion propagator is

$$\Delta_{\psi\alpha\dot\alpha}^{AB} = \frac{p_\mu\sigma^\mu_{\alpha\dot\alpha}}{p^2}P^{AB}, \qquad (2.14)$$

where the momentum enters at the end of the propagator with the undotted index, and the auxiliary propagator is

$$\Delta_F^{AB} = P^{AB}. \qquad (2.15)$$

3 Renormalisation

The bare action will be given as usual by replacing fields and couplings by their bare versions, shortly to be given more explicitly. Note that in the $\mathcal{N} = \frac{1}{2}$ supersymmetric case, fields and their conjugates may renormalise differently. We found in refs. [19, 20] that non-linear renormalisations of λ and $\overline{F}$ were required; and in a subsequent paper [34] we pointed out that non-linear renormalisations of F, $\overline{F}$ are required even in ordinary $\mathcal{N} = 1$ supersymmetric gauge theory when working in the uneliminated formalism. The renormalisations of the remaining fields and couplings are linear as usual (except for κ, κ_{1-5}, see later) and given by

$$
\begin{aligned}
&\overline{\lambda}_B^a = Z_\lambda^{\frac{1}{2}} \overline{\lambda}^a, && A_{\mu B}^a = Z_A^{\frac{1}{2}} A_\mu^a, && \phi_B^a = Z_\phi^{\frac{1}{2}} \phi^a, && \psi_B^a = Z_\psi^{\frac{1}{2}} \psi^a, \\
&\overline{\phi}_B^a = Z_\phi^{\frac{1}{2}} \overline{\phi}^a, && \overline{\psi}_B^a = Z_\psi^{\frac{1}{2}} \overline{\psi}^a, && g_B = Z_g g, && y_B = Z_y y, \\
&C_B^{\mu\nu} = Z_C C^{\mu\nu}, && (\kappa - 1)_B = Z_\kappa (\kappa - 1), && \kappa_{1-5 B} = Z_{1-5}.
\end{aligned}
\tag{3.1}
$$

The corresponding U(1) gauge multiplet fields $\overline{\lambda}^0$ etc are unrenormalised; so is g_0. The renormalisation constants for the U(1) chiral fields will be denoted Z_{ϕ^0} etc and discussed later. In eq. (3.1), Z_{1-5} are divergent contributions; in other words we have set the renormalised couplings κ_{1-5} to zero for simplicity. The anomalous dimensions Z_λ etc, and the renormalisation constants for the couplings g, y, C and $(\kappa - 1)$, start with tree-level values of 1. (The slightly non-standard definition of Z_κ is once again to make our results correspond more closely with those of ref. [33].) The one-loop graphs contributing to the "standard" terms in the Lagrangian (those without a $C^{\mu\nu}$) are the same as in the $\mathcal{N} = 1$ case, though we must now take into account the κ dependence of the propagators for the U(1) chiral fields, as seen in eqs. (2.12), (2.14) and (2.15); however, the anomalous dimensions for the gauge-multiplet fields and hence the gauge β-functions are the same as in the standard $\mathcal{N} = 1$ theory. Since our gauge-fixing term in eq. (2.10) does not preserve supersymmetry, the anomalous dimensions for A_μ^a and λ^a are different (and moreover gauge-parameter dependent), as are those for ϕ^a and ψ^a. However, the gauge β-functions are of course gauge-independent. We therefore have, at one loop [35]:

$$
\begin{aligned}
Z_\lambda &= 1 - 2g^2 N L(3 + \alpha), \\
Z_A &= 1 - g^2 N L(3 + \alpha), \\
Z_D &= 1 - 6g^2 N L, \\
Z_g &= 1 - 2g^2 N L,
\end{aligned}
\tag{3.2}
$$

where (using dimensional regularisation with $d = 4 - \epsilon$) $L = \frac{1}{16\pi^2 \epsilon}$; the results appear different from those in ref. [35] and indeed our earlier paper ref. [23] due to our absorption of the factor of g into the gauge multiplet fields.

The divergent contributions corresponding to (for instance) the scalar kinetic terms take the form

$$
L \left(-\mathrm{tr}[\tilde{D}^A P \tilde{D}^B P] y \overline{y} \partial^\mu \overline{\phi}^A \partial_\mu \phi^B + 2g^2 (1 - \alpha) \partial^\mu \overline{\phi}^a \partial_\mu \phi^a \right)
$$

$$= L \left\{ -y\overline{y} \left[N + \frac{4}{N\kappa}(1-\kappa) \right] + 2g^2(1-\alpha) \right\} \partial^\mu \overline{\phi}^a \partial_\mu \phi^a$$

$$-2Ly\overline{y} \left[N + \frac{1}{N\kappa^2}(1-\kappa^2) \right] \partial^\mu \overline{\phi}^0 \partial_\mu \phi^0 \tag{3.3}$$

and this must be cancelled by

$$- \left[Z_\phi \partial^\mu \overline{\phi}^a \partial_\mu \phi^a + Z_{\phi^0} \partial^\mu \overline{\phi}^0 \partial_\mu \phi^0 + Z_\kappa(\kappa - 1) Z_{\phi^0} \partial^\mu \overline{\phi}^0 \partial_\mu \phi^0 \right]. \tag{3.4}$$

(Here and elsewhere, when we mention divergent contributions, we mean divergent contributions to the effective action.) We immediately find (using similar results for the fermion and auxiliary kinetic terms)

$$Z_\phi = \left\{ -y\overline{y} \left[N + \frac{4}{N\kappa}(1-\kappa) \right] + 2g^2(1-\alpha)N \right\} L,$$

$$Z_\psi = \left\{ -y\overline{y} \left[N + \frac{4}{N\kappa}(1-\kappa) \right] - 2g^2(1+\alpha)N \right\} L,$$

$$Z_F = -y\overline{y} \left[N + \frac{4}{N\kappa}(1-\kappa) \right] L. \tag{3.5}$$

The assignment of Z_{ϕ^0} (and Z_{ψ^0}, Z_{F^0}) requires more care (and note we are still at liberty to choose Z_κ). Consider the $yd^{abc}\phi^a\psi^b\psi^c$ term. The only diagrams contributing to this are gauge dependent and give (as usual)

$$-\frac{1}{2}(7+3\alpha)LNg^2 y d^{abc}\phi^a\psi^b\psi^c. \tag{3.6}$$

We then deduce that at one loop

$$Z_y^{(1)} = -\frac{1}{2}Z_\phi^{(1)} - Z_\psi^{(1)} - (7+3\alpha)LNg^2$$

$$= -\frac{3}{2} \left\{ -y\overline{y} \left[N + \frac{4}{N\kappa}(1-\kappa) \right] + 4g^2N \right\} L = -\frac{3}{2}Z_\Phi^{(1)}, \tag{3.7}$$

where we recognise

$$Z_\Phi^{(1)} = \left\{ -y\overline{y} \left[N + \frac{4}{N\kappa}(1-\kappa) \right] + 4g^2N \right\} L \tag{3.8}$$

as the one-loop contribution to the $SU(N)$ chiral superfield renormalisation constant. This is in accord with the non-renormalisation theorem. (We should note that the discussion of renormalisation of the $F\overline{\phi}^2$ and $\overline{F}\phi^2$ terms in the potential requires the non-linear renormalisations of $\overline{F}$, F which will be given explicitly later.) In the usual ($\kappa = 1$) case, the Yukawa terms involving (for instance) $\phi^0\psi^b\psi^c$ would renormalise differently from the $y\phi^a\psi^b\psi^c$ term due to the difference between Z_ϕ and Z_{ϕ^0}, and the different diagrams contributing to the two terms, and would need a different Yukawa coupling, y' say, for renormalisability. To be precise, we would have (in analogy with eq. (3.7), and again invoking the non-renormalisation theorem)

$$Z_{y'}^{(1)} = -\frac{1}{2}Z_{\Phi^0}^{(1)} - Z_\Phi^{(1)}. \tag{3.9}$$
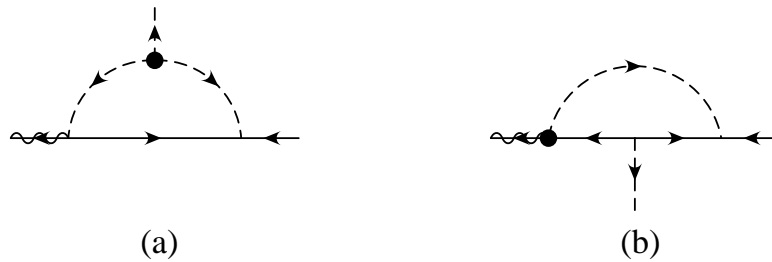
Figure 1. Diagrams with one gaugino, one scalar and one chiral fermion line (and two Yukawa couplings); the dot represents the position of a C.

On the other hand, the $\mathcal{N} = \frac{1}{2}$ supersymmetry transformations mix these two groups of terms and require them to have the same coupling. It therefore seems impossible to achieve simultaneously both renormalisability and $\mathcal{N} = \frac{1}{2}$ supersymmetry. The ingenious solution suggested in ref. [33] is to exploit the presence of κ to adjust Z_{Φ^0} to match Z_Φ. This then guarantees that y and y' may be identified. Moreover we note that the difference between Z_Φ and Z_ϕ, Z_ψ is due solely to the choice of a non-supersymmetric gauge; the gauge-independent terms are the same, and since there are no gauge interactions for the U(1) fields anyway, we have

$$Z_{\phi^0} = Z_{\psi^0} = Z_{\Phi^0}. \tag{3.10}$$

We then find from eqs. (3.3), (3.4), and (3.8)

$$Z_\kappa^{(1)} = -\frac{4g^2 N\kappa}{\kappa - 1} + \frac{y\overline{y}N(\kappa - 2)}{\kappa - 1} - \frac{2y\overline{y}(2\kappa^2 - \kappa - 1)}{N\kappa^2}. \tag{3.11}$$

We have now dealt with the majority of the renormalisations of fields and couplings. The remaining non-linear renormalisations of λ, F and $\overline{F}$ are largely determined in order to cancel C-dependent divergences; though as we have emphasised, a non-linear renormalisation of F and $\overline{F}$ is required in the usual $\mathcal{N} = 1$ ($C = 0$) case, and we shall quote the result of ref. [34]. So we now need to show how the C-dependent divergences are modified in the presence of κ and check that we can choose these non-linear renormalisations, together with κ_{1-5}, so that the theory is renormalisable. In particular we shall verify that with our choice of Z_κ and the identification of Z_{Φ^0} with Z_Φ, the full potential (which includes C-dependent terms) is indeed renormalisable with a single Yukawa coupling (though this is in principle guaranteed since the non-renormalisation theorem is known to extend to the $\mathcal{N} = \frac{1}{2}$ case [13]). The relevant divergent one-loop C-dependent graphs are depicted in figures 1–14. Figures 1–4 are graphs giving contributions proportional to $y\overline{y}$. Figures 1–3 were not computed by us previously in the adjoint case; we did compute figure 4, but in any case the result needs reassessing in the present case with our κ-dependent action, and will be radically different. Hence we shall shortly give a complete tabulation of the results for figures 1–4. Figures 5–14 were all computed previously and in fact we can obtain the results for our current κ-dependent action with very simple modifications. We shall therefore simply present the results.
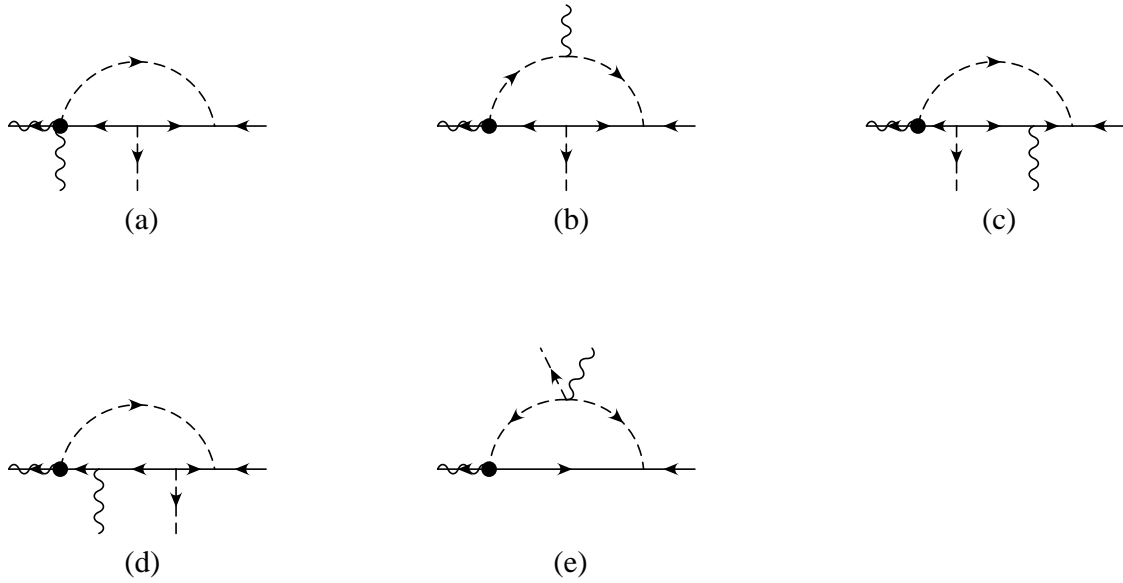
Figure 2. Diagrams with one gaugino, one scalar, one chiral fermion and one gauge line (and two Yukawa couplings).
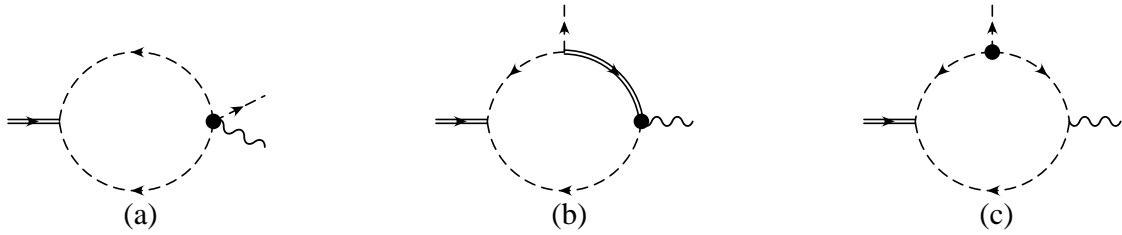
Figure 3. Diagrams with one auxiliary, one scalar and one gauge line (and two Yukawa couplings).

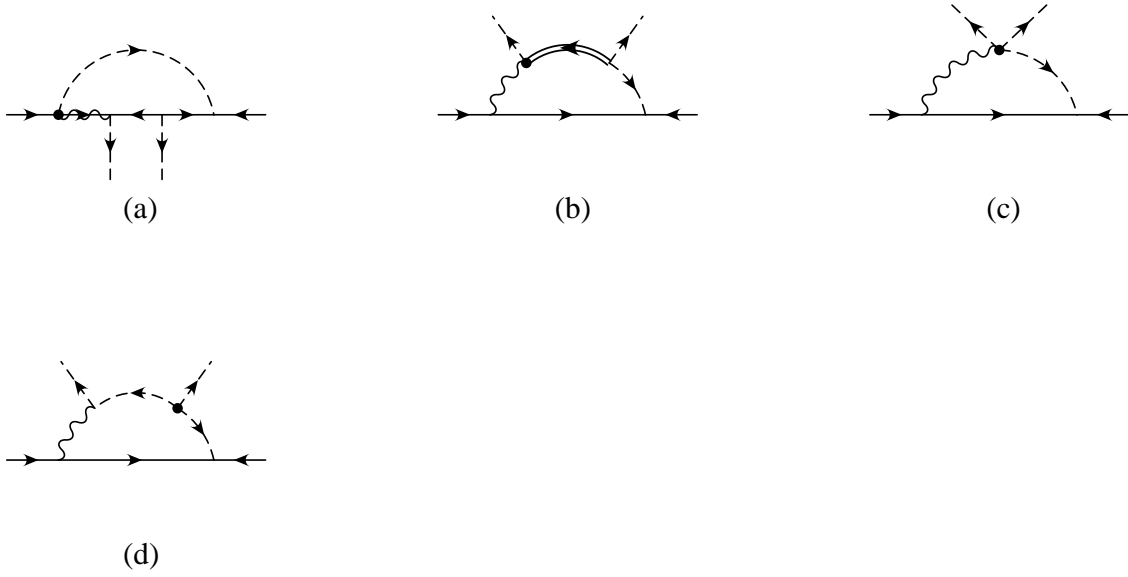|	X	X'
a	$\mathrm{tr}[\tilde{F}^A \tilde{F}^B \tilde{D}^C]$	0
b	0	$-\mathrm{tr}[\hat{D}^B P \tilde{D}^A P \tilde{D}^C P]$

Table 1. Divergent contributions from figure 1

The divergent contributions from figure 1 are of the form

$$\sqrt{2}C^{\mu\nu}y\overline{y}L(X^{ABC}\partial_\mu\overline{\phi}^A\overline{\lambda}^B\overline{\sigma}_\nu\psi^C + X'^{ABC}\overline{\phi}^A\overline{\lambda}^B\overline{\sigma}_\nu\partial_\mu\psi^C) \tag{3.12}$$

where X^{ABC}, X'^{ABC} are as given in table 1.

Here,

$$(\hat{D}^a)^{0b} = (\hat{D}^a)^{b0} = \kappa d^{ab0} \quad (\hat{D}^0)^{00} = \kappa d^{000},$$
$$(\hat{D}^A)^{BC} = d^{ABC} \quad \text{otherwise.} \tag{3.13}$$

Figure 4. Diagrams with two chiral fermion lines and two scalars (and two Yukawa couplings).

	Y
a	$2i\mathrm{tr}[\tilde{F}^A\hat{D}^B P\tilde{D}^C P\tilde{D}^D]$
b	$-i\mathrm{tr}[\tilde{F}^A\hat{D}^B P\tilde{D}^C P\tilde{D}^D]$
c	$-i\mathrm{tr}[\tilde{F}^A\tilde{D}^D P\hat{D}^B P\tilde{D}^C]$
d	0
e	$f^{ACX}\mathrm{tr}[\tilde{F}^X\tilde{F}^B\tilde{D}^D]$

Table 2. Divergent contributions from figure 2

Note that, although P derives from the chiral field propagators in eqs. (2.12), (2.14), (2.15), it is redundant when there is an F on either side.
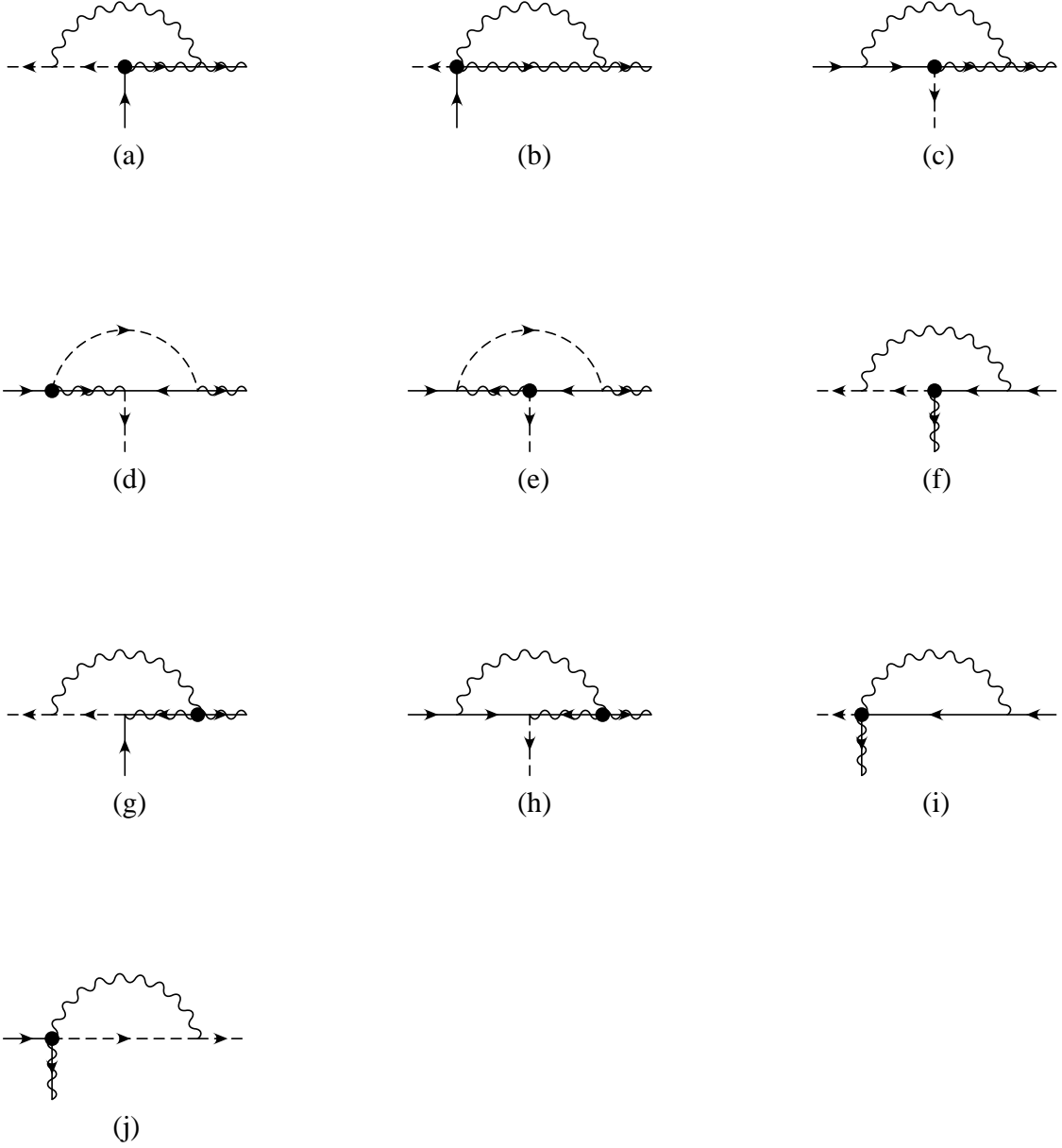
The divergent contributions from figure 2 are of the form

$$\sqrt{2}C^{\mu\nu}y\overline{y}LY^{ABCD}A_\mu^A\overline{\phi}^C\overline{\lambda}^B\overline{\sigma}_\nu\psi^D \tag{3.14}$$

where Y^{ABCD} is as given in table 2.

The contributions from figures 1, 2 add to

$$
\begin{aligned}
\Gamma_{1,2}^{(1)\mathrm{pole}} = {} & y\overline{y}LC^{\mu\nu}\Bigg\{ -\frac{1}{2}\left[N+\frac{8}{N\kappa}(1-\kappa)\right]d^{abc}\overline{\phi}^a\overline{\lambda}^b\overline{\sigma}_\nu D_\mu\psi^c + \frac{N}{2}D_\mu\overline{\phi}^a\overline{\lambda}^b\overline{\sigma}_\nu\psi^c \\
& -\left[N+\frac{4}{N\kappa}(1-\kappa)\right]d^{ab0}(\overline{\phi}^a\overline{\lambda}^0\overline{\sigma}_\nu D_\mu\psi^b + \overline{\phi}^0\overline{\lambda}^a\overline{\sigma}_\nu D_\mu\psi^b) \\
& -\left[N+\frac{2}{N\kappa^2}(1-\kappa^2)\right]d^{ab0}\overline{\phi}^a\overline{\lambda}^b\overline{\sigma}_\nu\partial_\mu\psi^0 + Nd^{ab0}D_\mu\overline{\phi}^a\overline{\lambda}^b\overline{\sigma}_\nu\psi^0, \\
& -2\left[N+\frac{1}{N\kappa^2}(1-\kappa^2)\right]d^{000}\overline{\phi}^0\overline{\lambda}^0\overline{\sigma}_\nu\partial_\mu\psi^0\Bigg\}
\end{aligned} \tag{3.15}
$$

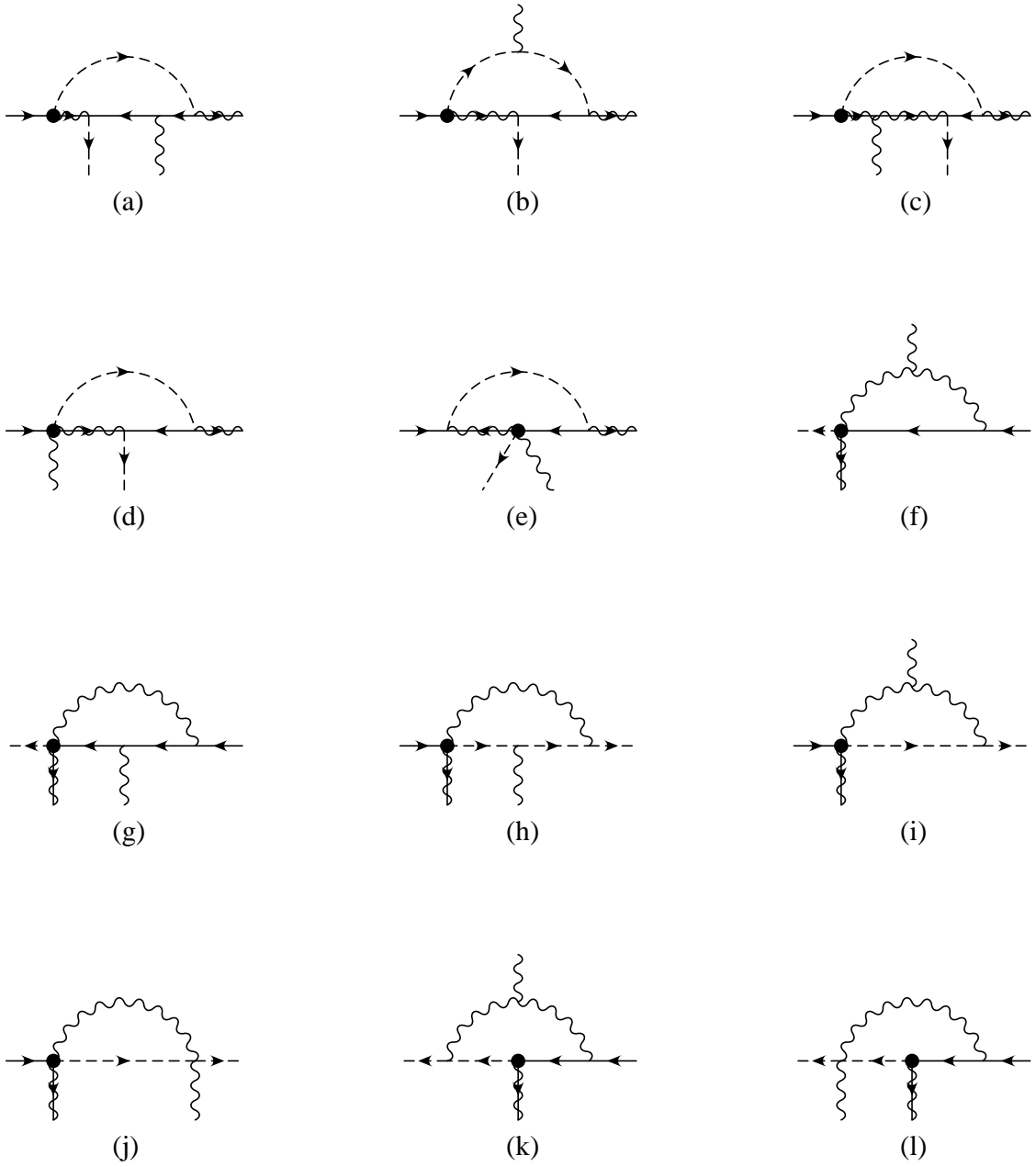**Figure 5**. Diagrams with one gaugino, one scalar and one chiral fermion line.

The divergent contributions from figure 3 are of the form

$$iC^{\mu\nu}y\overline{y}L(Z^{ABC}\partial_\mu\overline{\phi}^A A_\nu^B F^C + Z'^{ABC}\overline{\phi}^A\partial_\mu A_\nu^B F^C) \qquad (3.16)$$

where Z^{ABC}, Z'^{ABC} are as given in table 3. They add to

$$\Gamma_3^{(1)\text{pole}} = iy\overline{y}LC^{\mu\nu}\left[\frac{N}{2}d^{abc}\overline{\phi}^a F_{\mu\nu}^b F^c + Nd^{ab0}\overline{\phi}^a F_{\mu\nu}^b F^0\right], \qquad (3.17)$$

where we have assumed that the $\overline{\phi}AAF$ diagrams which we have not computed yield the
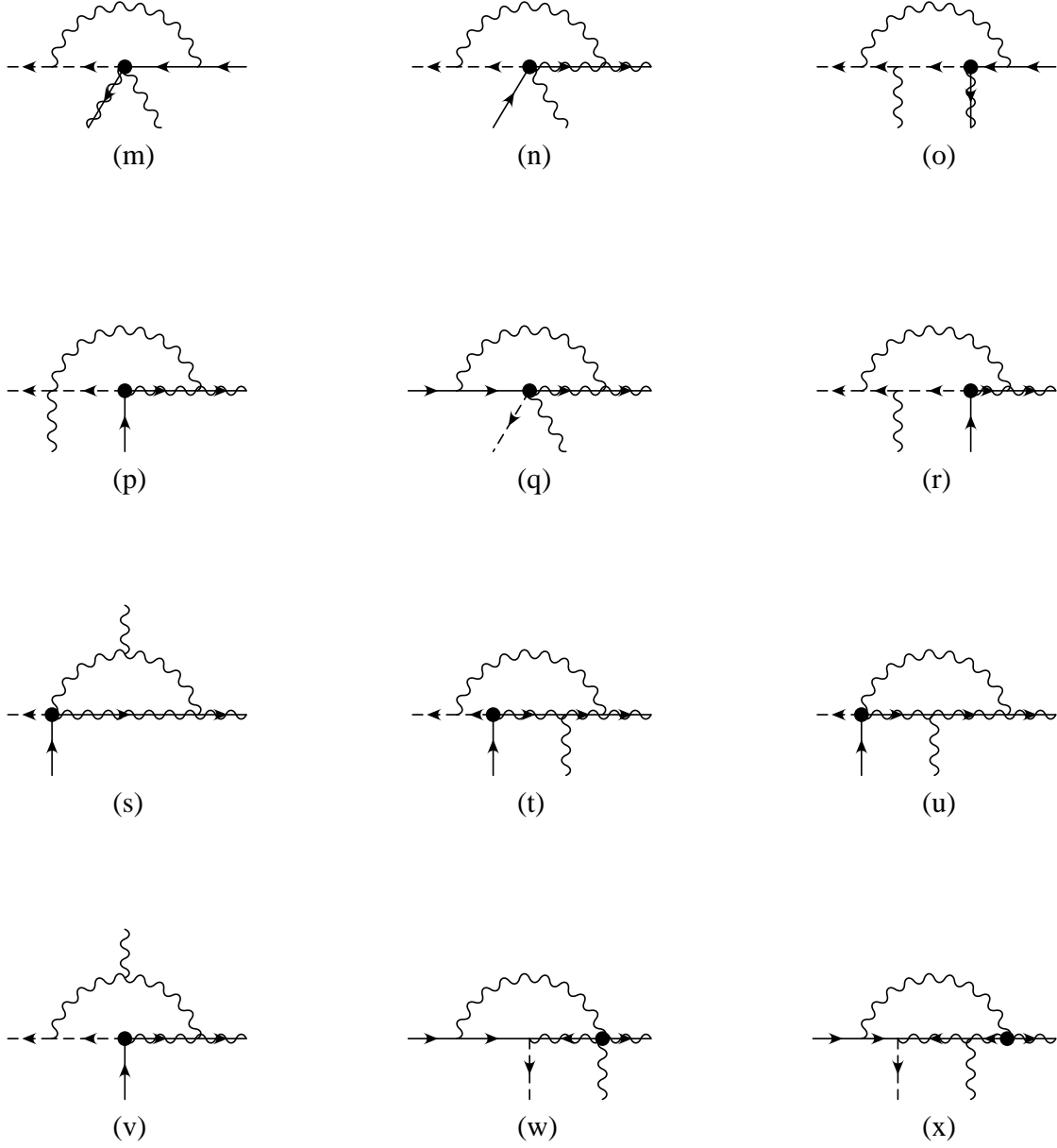
Figure 6. Diagrams with one gaugino, one scalar, one chiral fermion and one gauge line.

gauge completion of the $\overline{\phi}(\partial A)F$ terms. The contributions from figure 4 are given by

$$y\overline{y}g^2 L Z_1^{ABCD}(C\psi)^B \psi^A \overline{\phi}^C \overline{\phi}^D \tag{3.18}$$

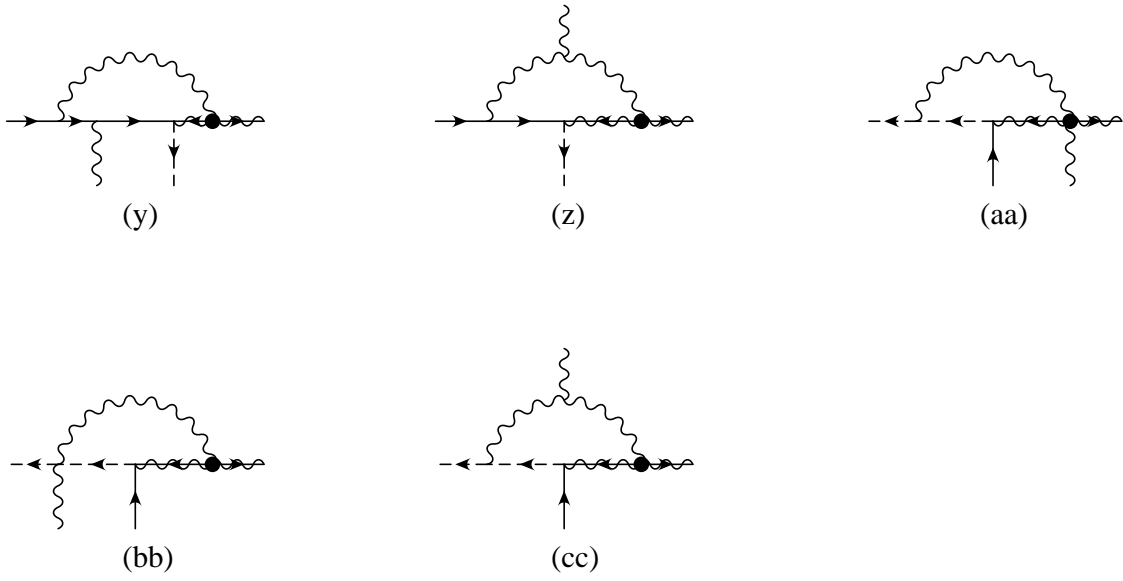where Z_1^{ABCD} is as given in table 4. They add to

$$\Gamma_4^{(1)\mathrm{pole}} = \left[N + \frac{4}{N\kappa}(1-\kappa) \right] y\overline{y}g^2 L \left[f^{abe} d^{cde}(C\psi)^b \psi^a \overline{\phi}^c \overline{\phi}^d + 2 f^{abe} d^{0ce}(C\psi)^b \psi^a \overline{\phi}^c \overline{\phi}^0 \right]. \tag{3.19}$$
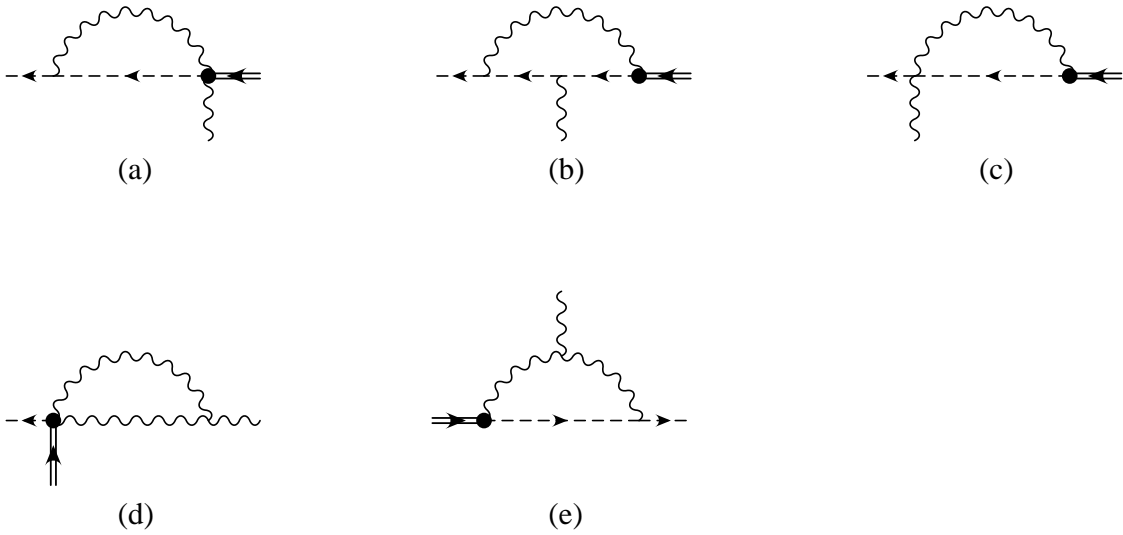
(m)

(n)

(o)

(p)

(q)

(r)

(s)

(t)

(u)

(v)

(w)

(x)

Figure 7. Diagrams with one gaugino, one scalar, one chiral fermion and one gauge line (continued).

|	Z	Z'
a	$2\mathrm{tr}[\tilde{F}^A\tilde{F}^B\tilde{D}^C]$	$\frac{2}{3}\mathrm{tr}[\tilde{F}^A\tilde{F}^B\tilde{D}^C] + \frac{8}{3}\mathrm{tr}[\tilde{D}^C P\tilde{D}^B\tilde{D}^A P]$ $+\frac{4}{3}\mathrm{tr}[\tilde{D}^C P\tilde{D}^D P]d^{DAB}$
b	0	$-4\mathrm{tr}[\tilde{D}^C P\hat{D}^B P\tilde{D}^A P]$
c	$-2\mathrm{tr}[\tilde{F}^A\tilde{F}^B\tilde{D}^C]$	0

Table 3. Divergent contributions from figure 3

– 12 –

(y)

(z)

(aa)

(bb)

(cc)

Figure 8. Diagrams with one gaugino, one scalar, one chiral fermion and one gauge line (continued).

(a)

(b)

(c)

(d)

(e)

Figure 9. Diagrams with one gauge, one scalar and one auxiliary line.

	Z_1
a	$-\mathrm{tr}[\tilde{F}^C \tilde{D}^B P \tilde{D}^A P \tilde{D}^D]$
b	$\mathrm{tr}[\tilde{F}^B \hat{D}^C P \tilde{D}^D \tilde{D}^A]$
c	$-\frac{1}{3}\left(\mathrm{tr}[\tilde{F}^B \tilde{D}^E P \tilde{D}^A]d^{CDE} + 2\mathrm{tr}[\tilde{F}^B \tilde{D}^C P \tilde{D}^D P \tilde{D}^A] - \mathrm{tr}[\tilde{F}^B \tilde{F}^C \tilde{F}^D \tilde{D}^A]\right)$
d	0

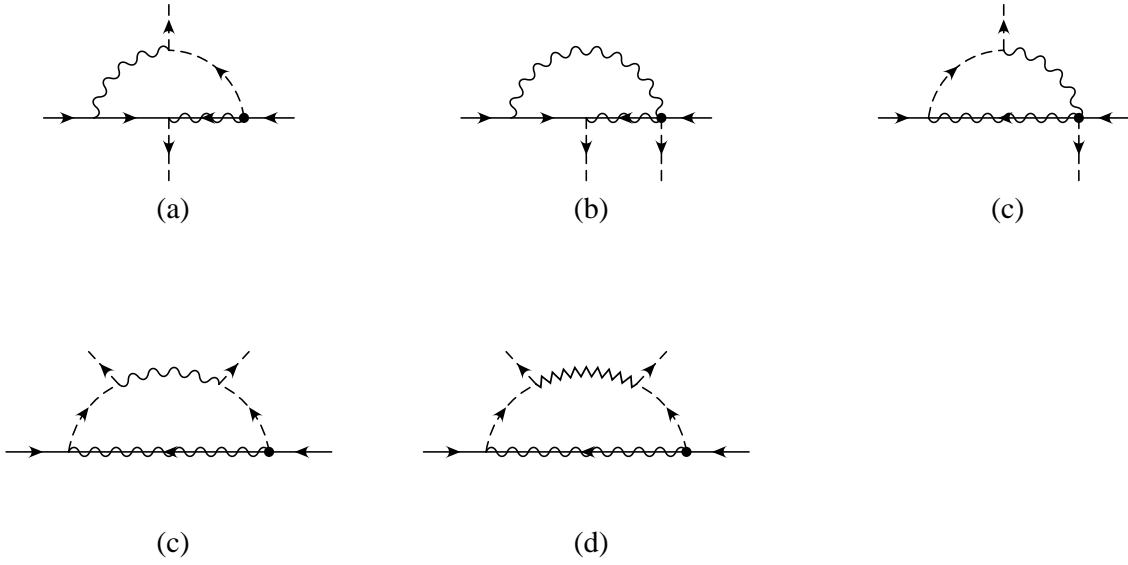**Table 4**. Divergent contributions from figure 4
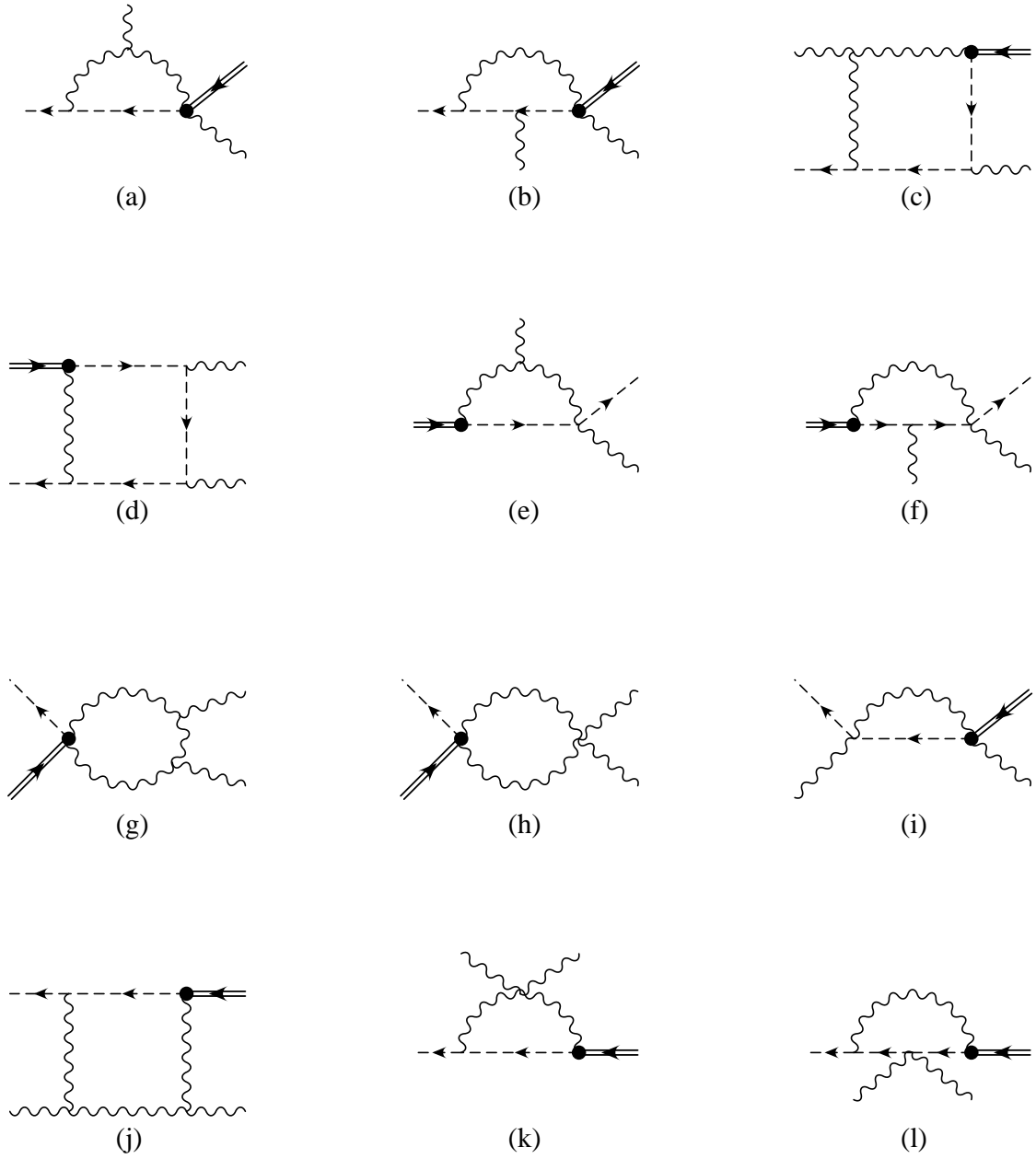
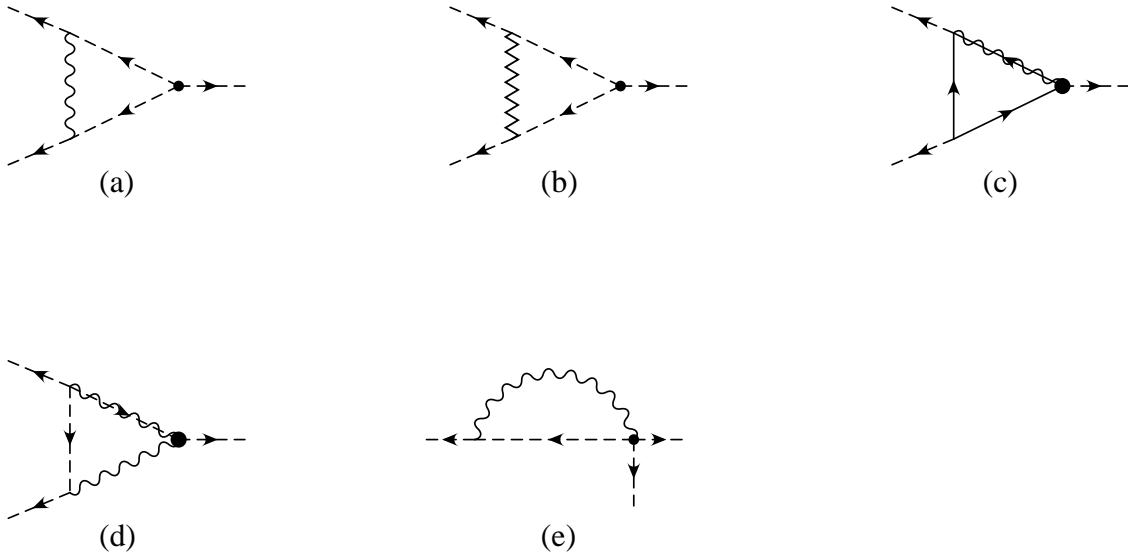**Figure 10**. Diagrams with two gauge, one scalar and one auxiliary line.

The contributions from figures 5–14 are listed below.

$$\Gamma_5^{(1)\text{pole}} = Ng^2\sqrt{2}LC^{\mu\nu}\Big[(2+3\alpha)d^{abc}\partial_\mu\overline{\phi}^a\overline{\lambda}^b\overline{\sigma}_\nu\psi^c - d^{abc}\overline{\phi}^a\overline{\lambda}^b\overline{\sigma}_\nu\partial_\mu\psi^c$$

$$+2\kappa(1+\alpha)d^{ab0}\partial_\mu\overline{\phi}^a\overline{\lambda}^b\overline{\sigma}_\nu\psi^0 - 2\kappa d^{ab0}\overline{\phi}^a\overline{\lambda}^b\overline{\sigma}_\nu\partial_\mu\psi^0$$

$$+2\alpha d^{ab0}\partial_\mu\overline{\phi}^a\overline{\lambda}^0\overline{\sigma}_\nu\psi^b$$

$$+2(1+\alpha)d^{ab0}\partial_\mu\overline{\phi}^0\overline{\lambda}^a\overline{\sigma}_\nu\psi^b\Big],$$
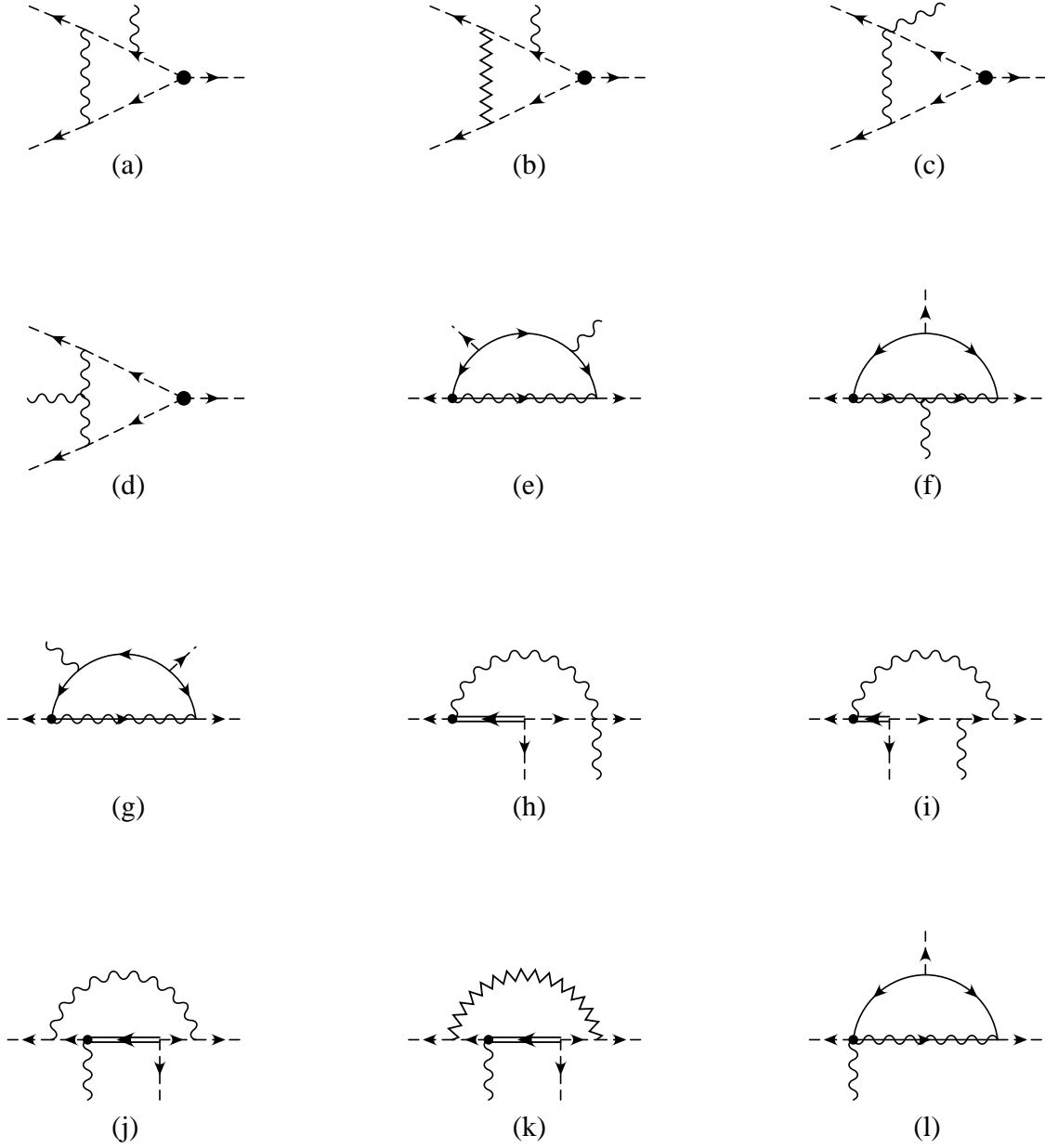
Figure 11. Diagrams with two scalar and two chiral fermion lines.

Figure 12. Diagrams with three scalar lines.

$$\Gamma_{6,7,8}^{(1)\text{pole}} = \sqrt{2}g^2 LC^{\mu\nu}A_\mu^a \left[\left(\frac{7}{2}(1+\alpha)f^{bae}d^{cde} - f^{dae}d^{cbe} + \frac{1}{2}f^{bde}d^{cae} \right) N\overline{\phi}^b\overline{\lambda}^c\overline{\sigma}_\nu\psi^d \right.$$

$$\left. -\frac{1}{2}(1+5\alpha)\sqrt{2N}f^{abc}\overline{\phi}^b\overline{\lambda}^0\overline{\sigma}_\nu\psi^c - \frac{1}{2}\kappa(7+5\alpha)\sqrt{2N}f^{abc}\overline{\phi}^b\overline{\lambda}^c\overline{\sigma}_\nu\psi^0 \right],$$

$$\Gamma_9^{(1)\text{pole}} = iNg^2 LC^{\mu\nu} \left[-(4-\alpha)d^{abc}\overline{\phi}^b\partial_\mu A_\nu^a F^c \right.$$

$$\left. -3\kappa(1-\alpha)d^{ab0}\overline{\phi}^a\partial_\mu A_\nu^b F^0 - (5+\alpha)d^{ab0}\overline{\phi}^0\partial_\mu A_\nu^a F^b \right],$$

Figure 13. Diagrams with three scalar, one gauge line.

$$\Gamma_{10}^{(1)\text{pole}} = ig^2 LC^{\mu\nu} A_\mu^a A_\nu^b \left(\frac{1}{4}(3-4\alpha) N f^{abe} d^{cde} \overline{\phi}^c F^d \right.$$

$$\left. -2\alpha\kappa\sqrt{2N} f^{abc} \overline{\phi}^c F^0 + \frac{3}{2}\sqrt{2N} f^{abc} \overline{\phi}^0 F^c \right),$$

$$\Gamma_{11}^{(1)\text{pole}} = -iLN g^2 C^{\alpha\beta} d^{abe} f^{cde} \overline{\phi}^a \overline{\phi}^b \psi_\alpha^c \psi_\beta^d,$$

$$\Gamma_{12}^{(1)\text{pole}} = \frac{1}{2} N\overline{y} g^2 LC^{\mu\nu}(1+\alpha) f^{abc} \partial_\mu \overline{\phi}^a \partial_\nu \overline{\phi}^b \overline{\phi}^c,$$
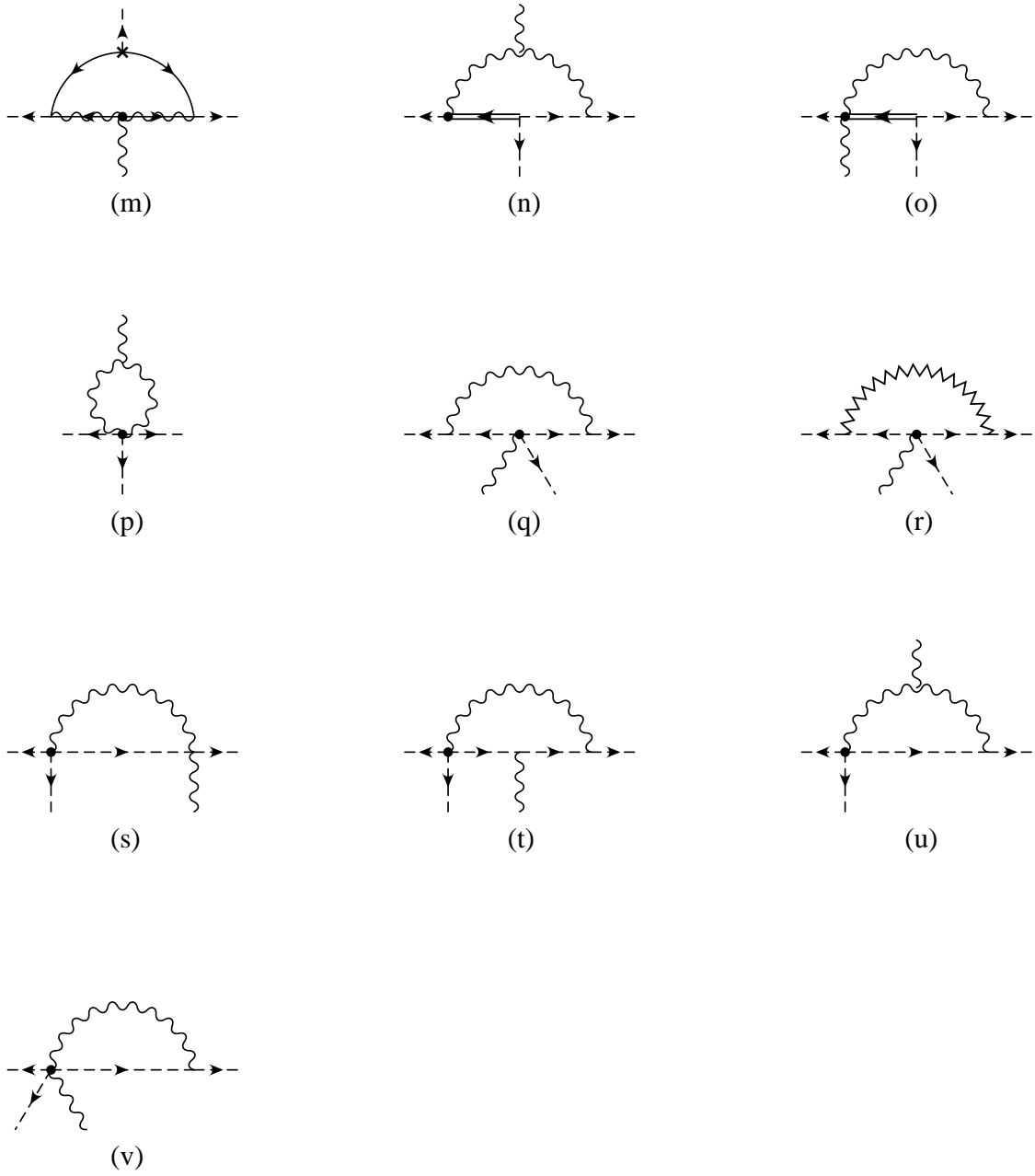
Figure 14. Diagrams with three scalar, one gauge line (continued)

$$\Gamma_{13,14}^{(1)\text{pole}} = iC^{\mu\nu}\overline{y}g^2 L\left(-\frac{1}{2}\left(3+\frac{7}{3}\alpha\right)Nf^{abe}f^{cde}\partial_\mu\overline{\phi}^a\overline{\phi}^b\overline{\phi}^c A_\nu^d\right.$$

$$\left. +\left[-\left(\frac{5}{4}-\frac{1}{6}\alpha\right)Nd^{abe}d^{cde}+\left(3+\frac{7}{3}\alpha\right)\delta^{ab}\delta^{cd}\right]\overline{\phi}^a\overline{\phi}^b\overline{\phi}^c\partial_\mu A_\nu^d$$

$$-\frac{1}{2}(9+\alpha)\sqrt{2N}d^{abc}\overline{\phi}^0\overline{\phi}^a\overline{\phi}^b\partial_\mu A_\nu^c - (5+\alpha)\overline{\phi}^0\overline{\phi}^0\overline{\phi}^a\partial_\mu A_\nu^a$$

$$-2\sqrt{2N}d^{abc}\overline{\phi}^a\overline{\phi}^b\overline{\phi}^c\partial_\mu A_\nu^0 - 8\overline{\phi}^a\overline{\phi}^a\overline{\phi}^0\partial_\mu A_\nu^0\bigg). \tag{3.20}$$

We now need to specify the remaining renormalisations, of F, $\overline{F}$ and λ, required to cancel the divergences. The renormalisation of λ^A is given by

$$\lambda_B^a = Z_\lambda^{\frac{1}{2}}\lambda^a - \frac{1}{2}NLg^2C^{\mu\nu}d^{abc}\sigma_\mu\overline{\lambda}^c A_\nu^b - NLg^2C^{\mu\nu}d^{ab0}\sigma_\mu\overline{\lambda}^0 A_\nu^b$$
$$+i\sqrt{2}\tau_1 NLg^4 d^{abc}(C\psi)^b\overline{\phi}^c + i\sqrt{2}\tau_2 NLg^4 d^{ab0}(C\psi)^0\overline{\phi}^b,$$
$$\lambda_B^0 = \lambda^0 + i\sqrt{2}\tau_3 NLg^2 g_0^2 d^{0ab}(C\psi)^a\overline{\phi}^b, \tag{3.21}$$

where $(C\psi)^\alpha = C^\alpha{}_\beta\psi^\beta$. The coefficients of the non-linear terms on the first line of eq. (3.21) were computed in ref. [20]; the values of τ_{1-3} will be specified later. The replacement of λ by λ_B produces a change in the action given (to first order) by

$$S_0(\lambda_B) - S_0(\lambda) = NLg^2 \int d^4x \bigg\{ -\frac{1}{2}f^{bde}d^{cae}A_\mu^a\overline{\phi}^b\overline{\lambda}^c\overline{\sigma}_\nu\psi^d - f^{abe}d^{ec0}A_\mu^a\overline{\phi}^b\overline{\lambda}^0\overline{\sigma}_\nu\psi^c$$
$$+\tau_1\big[ig^2 d^{abe}f^{cde}\overline{\phi}^a\overline{\phi}^b\psi^c(C\psi^d)$$
$$+\sqrt{2}C^{\mu\nu}d^{abc}\overline{\phi}^a\overline{\lambda}^b\overline{\sigma}_\nu D_\mu\psi^c + \sqrt{2}C^{\mu\nu}d^{abc}D_\mu\overline{\phi}^a\overline{\lambda}^b\overline{\sigma}_\nu\psi^c\big]$$
$$+\tau_2\sqrt{2}C^{\mu\nu}d^{ab0}(\overline{\phi}^a\overline{\lambda}^b\overline{\sigma}_\nu\partial_\mu\psi^0 + D_\mu\overline{\phi}^a\overline{\lambda}^b\overline{\sigma}_\nu\psi^0)$$
$$+\tau_3\sqrt{2}C^{\mu\nu}d^{0ab}(\overline{\phi}^a\overline{\lambda}^0\overline{\sigma}_\nu D_\mu\psi^b + D_\mu\overline{\phi}^a\overline{\lambda}^0\overline{\sigma}_\nu\psi^b) + \cdots \bigg\}, \tag{3.22}$$

where the ellipsis indicates terms depending solely on gauge or gaugino fields (which were given previously in ref. [20]).

We now find that to render finite the contributions linear in F, we also require

$$\overline{F}_B^a = Z_F\overline{F}^a + iC^{\mu\nu}Lg^2\bigg\{ N\bigg[(5+2\alpha)\partial_\mu A_\nu^b - \frac{1}{4}(11+4\alpha)f^{bde}A_\mu^d A_\nu^e\bigg]\overline{\phi}^c d^{abc}$$
$$+\sqrt{2N}\bigg[2(2+\alpha)\partial_\mu A_\nu^a - \frac{1}{2}(5+2\alpha)f^{abc}A_\mu^b A_\nu^c\bigg]\overline{\phi}^0$$
$$+2\sqrt{2N}(3+\alpha)\partial_\mu A_\nu^0\overline{\phi}^a\bigg\} + (\alpha+3)g^2NL\frac{1}{4}yd^{abc}\phi^b\phi^c$$
$$+\frac{1}{2}(\alpha+3)yg^2NLd^{ab0}\phi^b\phi^0 + \tau_4 g^2 yLf^{abc}(C\psi)^b\psi^c + \cdots, \tag{3.23a}$$
$$\overline{F}_B^0 = Z_F\overline{F}^0 \tag{3.23b}$$
$$F_B^a = Z_F F^a + (\alpha+3)g^2NL\frac{1}{4}\overline{y}d^{abc}\overline{\phi}^b\overline{\phi}^c + \frac{1}{2}(\alpha+3)\overline{y}g^2NLd^{ab0}\overline{\phi}^b\overline{\phi}^0 + \cdots, \tag{3.23c}$$
$$F_B^0 = Z_F F^0, \tag{3.23d}$$

where the ellipsis stands for $\overline{\phi\lambda\lambda}$ terms which only affect the separately $\mathcal{N} = \frac{1}{2}$ independent terms which we are omitting anyway. We should mention here that in eq. (5.5) of ref. [23]

the $y\phi\phi$ and $\overline{y}\,\overline{\phi}\,\overline{\phi}$ terms in eq. (3.23a), (3.23c) were inadvertently interchanged. Writing $Z_C^{(n)}$ for the n-loop contribution to Z_C, and so on, we set

$$Z_n^{(1)} = z_n L. \tag{3.24}$$

We now find that with

$$
\begin{aligned}
z_C = 0, \quad &\tau_1 = 1, \quad \tau_2 = -2, \quad \tau_3 = 4, \\
&z_1 = \frac{1}{2}\left[N + \frac{8}{N\kappa}(1-\kappa)\right] y\overline{y}, \\
&z_2 = \left[N + \frac{4}{N\kappa}(1-\kappa)\right] y\overline{y}, \\
&z_3 = \left[N + \frac{2}{N\kappa^2}(1-\kappa^2)\right] y\overline{y} + 4g^2, \\
&z_4 = \left[N + \frac{4}{N\kappa}(1-\kappa)\right] y\overline{y} - 4g^2, \\
&z_5 = 2\left[N + \frac{1}{N\kappa^2}(1-\kappa^2)\right] y\overline{y}, \\
&\tau_4 = \left[N + \frac{2}{N\kappa^2}(1-\kappa^2)\right].
\end{aligned}
\tag{3.25}
$$

the one-loop effective action is finite. In particular, the same coupling y is sufficient for the renormalisation of the full set of potential terms; and also the same non-anticommutativity parameter $C^{\mu\nu}$ is sufficient throughout and remains unrenormalised at one loop. This is in contrast to the situation in ref. [23], where we were obliged to introduce several different Yukawa couplings and also different $C^{\mu\nu}$ parameters for different groups of terms.

We note that the groups of terms involving κ_{1-5} have an analogue in ref. [33], in the groups of terms involving (in their notation) t_{1-5}, each group again being separately invariant. Explicit one-loop results are not given for t_{1-5}; in any case, we should probably not expect precise agreement due to our different gauge choices. While on the topic of comparison of the component and superfield approaches, we should mention the calculation of ref. [36]. There a three-field U(1) $\mathcal{N} = \frac{1}{2}$ model is considered in the superfield context. However, there the chiral fields are in the adjoint representation, whereas in ref. [32] we considered a three-field U(1) $\mathcal{N} = \frac{1}{2}$ model with the chiral fields having charges q, $-q$, 0. At least as far as the non-gauge parts of the results are concerned, we appear to have agreement.

4 Conclusions

We have confirmed by a component calculation the conclusion reached in ref. [33], namely that the general SU(N) $\otimes$ U(1) $\mathcal{N} = \frac{1}{2}$ theory with a superpotential may be rendered renormalisable by a judicious choice of kinetic term for the U(1) fields such that the renormalisations of the U(1) and SU(N) chiral superfields are equal, which ensures that a single Yukawa coupling is sufficient. This solves the difficulties which we encountered in ref. [23]; apart from restoring renormalisability, we also are no longer obliged to introduce several

different non-anticommutativity tensors $C^{\mu\nu}$, some of which require a non-zero renormalisation. $C^{\mu\nu}$ is unrenormalised at one loop. Our component calculation is perhaps technically simpler than the superfield one (though of course the brevity of the current paper owes much to our exploitation of previous results in ref. [23], and the fact that we have not computed divergences corresponding to separately $\mathcal{N} = \frac{1}{2}$ invariant terms). However, this is offset by the awkwardness of the various non-linear renormalisations which are required. We should mention that we have checked that the computation can also be carried out in the eliminated formalism, i.e. after eliminating F, $\overline{F}$ using their equations of motion.

Since $C^{\mu\nu}$ is now confirmed to be completely unrenormalised at one loop, it seems to us that the most pressing direction for further investigation is to see whether this property extends to two loops. However, $C^{\mu\nu}$ being a self-dual tensor, problems concerned with extending the definition of the alternating tensor $\epsilon^{\mu\nu\rho\sigma}$ away from four dimensions seem likely to arise when using dimensional regularisation beyond one loop. A promising alternative could be the use of differential regularisation [37].

A Group identities

Identities for SU(N) useful for simplifying the divergent contributions listed in the tables are [38]

$$\mathrm{tr}[\tilde{D}^a \tilde{D}^b] = \frac{N^2 - 4}{N} \delta^{ab}, \qquad\qquad \mathrm{tr}[\tilde{D}^a \tilde{D}^b \tilde{D}^c] = \frac{N^2 - 12}{2N} d^{abc},$$

$$\mathrm{tr}[\tilde{F}^a \tilde{F}^b \tilde{D}^c] = \frac{N}{2} d^{abc}, \qquad\qquad \mathrm{tr}[\tilde{F}^a \tilde{D}^b \tilde{D}^c] = i\frac{N^2 - 4}{2N} f^{abc},$$

$$\mathrm{tr}[\tilde{F}^a \tilde{F}^b \tilde{F}^c \tilde{D}^d] = i\frac{N}{4}(d^{abx} f^{cdx} + d^{cdx} f^{abx}),$$

$$\mathrm{tr}[\tilde{F}^a \tilde{D}^b \tilde{D}^c \tilde{D}^d] = \frac{N^2 - 12}{4N} f^{abx} d^{cdx} + \frac{N}{4} d^{abx} f^{cdx}$$
$$+ \frac{1}{N}(f^{adx} d^{cbx} - f^{acx} d^{bdx}). \tag{A.1}$$

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